

# Deprojecting Densities from Angular Cross-Correlations

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## ABSTRACT

I present a model-independent spherically symmetric density estimator to be used in the cross-correlation of imaging catalogs with objects of known redshift. The estimator is a simple modification of the usual projected density estimator, with weightings that produce a spherical aperture rather than a cylindrical one.

*Subject headings:* large-scale structure of the universe — methods: statistical

## 1. Introduction

Measuring galaxy properties as a function of their local environment is a central task of observational extragalactic astronomy. Doing so requires a measure of density. With redshift surveys, one can estimate densities by counting other spectroscopic galaxies in a redshift-space window around each primary object. However, imaging catalogs are generally much deeper than spectroscopic catalogs, which suggests that cross-correlating the imaging catalog with the spectroscopic catalog should yield useful densities even for the faintest spectroscopic objects.

The presence of at least one spectroscopic redshift in each pair is of great utility because it means that one can immediately map angular separations to physical distances and derive intrinsic properties (i.e. luminosities) for both the spectroscopic object and the correlated objects from the imaging catalog. This opportunity has been used extensively in the early study of galaxy clustering (Davis et al. 1978; Yee & Green 1987; Lilje & Efstathiou 1988; Saunders et al. 1992) and in the study of dwarf galaxies in groups (e.g., [Fer91] and in the field (Phillipps & Shanks 1987; Vader & Sandage 1991; Lorrimer et al. 1994; Loveday 1997). Of course, the correlated objects are still seen in projection, with a mix of spatial separations at each transverse separation.

Angular correlations can be inverted to spatial correlations, and hence to a form of density, by the assumption of isotropy, and many of the above studies did this by assuming that the correlation functions are power laws in scale (e.g., [Phi85]. Saunders et al. (1992) and Loveday (1997) go one step further by using the assumed power-law as an optimal filter. Fall & Tremaine (1977) suggest that the general inversion can be done with a smoothing filter. Baugh & Efstathiou (1993) use a regularized Lucy’s iteration to do a similar inversion, while Dodelson & Gaztañaga (2000) and Eisenstein & Zaldarriaga (2001) use other smoothing priors.

In the study of galaxy environmental dependences on small scales, it is useful to pursue a more model-independent measure of the density. On scales of 1 Mpc, it is quite possible that the correlation functions will deviate from power laws (and certainly from a uniform power law) in ways that depend on luminosity, star-formation rate, or other variables (e.g., [Scr02].

Here I describe a method to recover density estimates in spherically symmetric real-space windows from the cross-correlation of imaging and spectroscopic catalogs independently of the shape of the correlation function. The resulting formulae are a simple alteration of the usual background subtraction methods. The method can be subdivided so as to yield noisy density estimates for individual spectroscopic objects that can then be averaged according to whatever subclasses one might desire.

## 2. Angular Cross-correlations

### 2.1. Definitions

We begin with two samples of objects, one with spectroscopic redshifts and the other without. We bin the spectroscopic sample into thin redshift bins (indeed, we can consider each spectroscopic object separately) and select a subsample of the imaging catalog using the known redshift. For example, one might select a sample in a particular luminosity range (i.e. the objects would have the desired luminosity were they at the spectroscopic redshift). We adopt a flat-sky coordinate system in which the transverse directions are measured as distance  $\vec{R}$  at the given redshift and the linear direction is measured as distance  $Z$  where  $Z = 0$  is at the redshift. We write the three-dimensional position  $(\vec{R}, Z)$  as  $\vec{r}$ .

Of course, the imaging subsample samples a range of  $Z$ , not just  $Z = 0$ . We describe the homogeneous density of the sample as  $\phi(Z)$ , where this is the number per unit  $Z$  and per unit transverse area. The areal number density is

$$\bar{n} = \int_{-\infty}^{\infty} dZ \phi(Z). \quad (1)$$

If the true spatial cross-correlation between the spectroscopic object and the imaging subsample is  $\xi_{\text{is}}(r)$ , then the angular correlation is

$$w_{\text{is}}(R) = \frac{1}{\bar{n}} \int_{-\infty}^{\infty} dZ \phi(Z) \xi_{\text{is}}(\sqrt{Z^2 + R^2}) \approx \frac{2\phi_0}{\bar{n}} \int_0^{\infty} dZ \xi_{\text{is}}(\sqrt{Z^2 + R^2}) \quad (2)$$

where the last equality assumes that the selection function  $\phi$  is essentially constant over the scale on which  $\xi_{\text{is}}$  is not negligible. We also assume that the angular diameter distance is constant in this region. We denote  $\phi(0)$  as  $\phi_0$ . Equation (2) is a simple form of Limber's equation (Limber 1953; Groth & Peebles 1977).

### 2.2. Deprojection

It is well-known that equation (2) can be inverted as an Abel integral (von Zeipel 1908), so that

$$\xi_{\text{is}}(r) = -\frac{\bar{n}}{\phi_0 \pi} \int_r^{\infty} \frac{dR}{\sqrt{R^2 - r^2}} \frac{dw_{\text{is}}(R)}{dR}. \quad (3)$$

However, because this involves a derivative of the measured  $w_{\text{is}}(R)$ , it is noisy.

We address this by measuring only an integral of  $\xi_{\text{is}}$

$$\Delta = \frac{1}{V} \int_0^{\infty} 4\pi r^2 dr \xi_{\text{is}}(r) W(r) \quad (4)$$

where  $V = \int_0^{\infty} 4\pi r^2 dr W(r)$ .  $W(r)$  is our smoothing window.  $\Delta$  has a very useful physical meaning: it is the average overdensity of objects from the imaging catalog in the neighborhood (as defined by  $W$ ) of a

spectroscopic object. Note that  $\Delta$  is defined in 3-space with a spherically symmetric window; it is *not* a projected quantity.

Inserting (3) into (4) and switching the limits of integration yields

$$\Delta = -\frac{\bar{n}}{\phi_0 V \pi} \int_0^\infty dR \frac{dw_{\text{is}}}{dR} \int_0^R dr \frac{4\pi r^2 W(r)}{\sqrt{R^2 - r^2}} = -\frac{2\pi\bar{n}}{\phi_0 V} \int_0^\infty dR \frac{dw_{\text{is}}}{dR} F(R) \quad (5)$$

where

$$F(R) = \frac{2}{\pi} \int_0^R dr \frac{r^2 W(r)}{\sqrt{R^2 - r^2}}. \quad (6)$$

For bounded  $W(r)$ ,  $F(0) = 0$  and  $F(R) \rightarrow V/2\pi^2 R$  for large  $R$ . Integrating by parts yields

$$\Delta = \frac{\bar{n}}{\phi_0 V} \int_0^\infty 2\pi R dR w_{\text{is}}(R) G(R) \quad (7)$$

$$G(R) \equiv \frac{1}{R} \frac{dF}{dR}. \quad (8)$$

A constant  $w_{\text{is}}$  integrates to  $\Delta = 0$ , so adding a constant to  $w_{\text{is}}$  doesn't change  $\Delta$ .  $G(R) \rightarrow -V/2\pi^2 R^3$  for large  $R$ .

Now let us consider our measurement of  $w_{\text{is}}(R)$ . In a small radial bin from  $R$  to  $R + dR$ , we would estimate  $1 + w_{\text{is}}(R)$  as the ratio of the observed counts of pairs in that radial range to the expected number. The expected number is  $2\pi\bar{n}R dR$ . Treating the integral as a Riemann sum in which the bins are so small as to contain 0 or 1 observed pair leads to the conclusion that

$$\Delta = \frac{1}{N_{sp}} \sum_{j \in \{sp\}} \frac{1}{\phi_0 V} \sum_{k \in \{im\}} G(R_{jk}) \quad (9)$$

where the sums are over the objects in spectroscopic subsample and the imaging subsample, respectively,  $N_{sp}$  is the number of spectroscopic objects, and  $R_{jk}$  is the transverse separation of the  $j^{th}$  spectroscopic object to the  $k^{th}$  imaging object.

An important point is that we can now treat each spectroscopic object separately, yielding the following noisy measure of the overdensity around object  $j$ :

$$\Delta_j = \frac{1}{\phi_0 V} \sum_{k \in \{im\}} G(R_{jk}). \quad (10)$$

Note how simple this formula is: one counts the imaging objects, weighting by  $G(R)$ , and divides by the expected number of objects in the real-space window ( $V\phi_0$ ). We can recover the average density around any subset of the spectroscopic sample simply by averaging the selected  $\Delta_j$ .

It is interesting to compare equation (10) to a more conventional background-subtraction method in which one sums all of the objects in an angular aperture and subtracts an appropriately scaled value of the areal density averaged over the entire survey. This would correspond to a  $G$  function that was constant and positive for  $R$  less than the aperture and then constant and negative for all greater  $R$ . The difference is that this background subtraction would give an estimate for the density in a cylindrical region, in which the axis of the cylinder lies along the line of sight and is much longer than the radius of the cylinder. The resulting density would sample the correlation function  $\xi_{\text{is}}$  at a wide range of radii. The formula given here creates a compact region in all three dimensions. Some workers (e.g., [Gai97, Val97]) have used annular regions for the determination of the background. This truncates the cylindrical region in some fashion, but the detailed effects were not assessed.

### 2.3. Gaussian Windows

For a useful and illustrative example, we will treat the case in which the window is assumed to be a Gaussian  $W(r) = \exp(-r^2/2a^2)$ . The volume of the window is  $V = (2\pi)^{3/2}a^3$ . Then

$$F(R) = \frac{2}{\pi} \int_0^R dr \frac{r^2 e^{-r^2/2a^2}}{\sqrt{R^2 - r^2}} = \frac{R^2}{2} e^{-s} [I_0(s) - I_1(s)] \quad (11)$$

where  $s = R^2/4a^2$  and  $I_n$  are modified Bessel functions of the first kind (Gradshteyn & Ryzhik 2000, 3.364.1). Then

$$G(R) = e^{-s} [I_0(s) - 2sI_0(s) + 2sI_1(s)]. \quad (12)$$

The weighting function  $G$  is shown in Figure 1. The function is smooth, with a positive peak at  $R \approx a$  and a broader negative peak at  $R \approx 3a$ . As expected, to estimate the density averaged over the window  $W$ , one is counting the nearby objects and subtracting a count of objects slightly further away.

### 2.4. Boundaries and Masks

Equation (7) requires an integration over all radii and since  $G \propto R^{-3}$  at large radius, this integration converges as  $\sim 1/R$ . If one splits the integral in equation (7) at some radius  $R_{\max}$ , inside of which one will do the counting in equation (9), then the residual error from larger radii is

$$\Delta_{res} = \frac{\bar{n}}{\phi_0 V} \int_{R_{\max}}^{\infty} 2\pi R dR [1 + w_{is}(R)] G(R) \quad (13)$$

Here, we have to include the constant background term because it will no longer integrate to zero. We can estimate the correlated term by treating  $w_{is}$  as a power-law  $w_{is}(R) = w_{is}(R_{\max})(R_{\max}/R)^\alpha$ , where  $w_{is}(R_{\max})$  is the value at the truncation radius and  $\alpha$  is the slope of the power-law, typically 0.7–0.8 in past measurements. Then, using  $G(R) \approx -V/2\pi^2 R^3$  in the second term, we find

$$\Delta_{res} = -\frac{\bar{n}}{\phi_0} \left[ \frac{2\pi F(R_{\max})}{V} + \frac{w_{is}(R_{\max})}{\pi R_{\max}} \frac{1}{1 + \alpha} \right]. \quad (14)$$

Taking  $\xi_{is} \propto r^{-1-\alpha}$ , we have

$$\frac{\bar{n}}{\phi_0} w_{is}(R) = R \xi_{is}(R) \frac{\Gamma(\frac{\alpha}{2})\sqrt{\pi}}{\Gamma(\frac{\alpha+1}{2})} \quad (15)$$

One finds that  $\Delta_{res} = -2\pi\bar{n}F(R_{\max})/\phi_0 V - 0.70\xi_{is}(R_{\max})$  for  $\alpha = 0.75$ . Applying this as a correction does introduce some model dependence on the form of  $w_{is}$ , but one can pick  $R_{\max}$  so that the correction is rather small. In this way, one can avoid summing over all pairs of spectroscopic and imaging objects.

The circular region  $R < R_{\max}$  may still include regions that are outside the survey or its mask. If one writes  $\Phi(R)$  as the fraction of the annulus of radius  $R$  that is within the survey, then one can weight the counts in equation (9) by  $1/\Phi(R)$ . However,  $\Phi(R)$  may be expensive to compute for each spectroscopic galaxy. An alternative method is to generate a catalog of random points that are uniformly distributed *outside* of the survey region and then add to equation (10) the sum over those points closer than  $R_{\max}$ , weighting by  $G(R)[1 + w_{is}(R)]$  and the ratio of  $\bar{n}$  to the random catalog surface density. This effectively interpolates over the masked regions. If one treats  $w_{is}$  as a function of fixed shape but unknown amplitude, then one can do the sum of the clustered term separately and renormalize after one has determined the amplitude from the co-added  $\Delta_j$ . Of course, since one is in some fashion assuming the answer, one should exclude spectroscopic galaxies for which the masked contribution is large.

## 2.5. Errors and Optimal Weighting

When averaging the individual estimates  $\Delta_j$  from a sample of spectroscopic objects, one can achieve more precision by using non-uniform weighting in the mean. The optimal weighting is inverse variance; however, one should remember that approximations to inverse variance may be much simpler to compute and yet still close to optimal. One should never weight by variances that are derived from single data points, e.g. the square root of the observed number of companions to a given galaxy, because this will bias the resulting mean.

The statistical variance of the  $\Delta_j$  estimator comes from two sources, the clustering of the galaxies and shot noise. The clustering term involves the three-point correlation function, with one galaxy from the spectroscopic sample and two from the imaging sample. If the first of these is at the origin and the latter two are at  $\vec{r}_1$  and  $\vec{r}_2$ , then we denote the three-point correlation function as  $\zeta(\vec{r}_1, \vec{r}_2)$ . The expected densities at two points given a spectroscopic galaxy at the origin is (Peebles 1980)

$$\langle \rho(\vec{r}_1) \rho(\vec{r}_2) \rangle = \phi(Z_1) \phi(Z_2) [1 + \xi_{\text{is}}(r_1) + \xi_{\text{is}}(r_2) + \xi_{ii}(|\vec{r}_1 - \vec{r}_2|) + \zeta(\vec{r}_1, \vec{r}_2)] \quad (16)$$

where  $\xi_{\text{is}}$  is the two-point correlation between imaging and spectroscopic galaxies and  $\xi_{ii}$  is the correlation of two imaging galaxies.

The clustering contribution to the variance of  $\Delta_j$  is then

$$V_{\text{cl}}(\Delta_j) = \int d^2 R_1 \int d^2 R_2 G(R_1) G(R_2) \int dZ_1 dZ_2 \times \\ [\xi_{ii}(|\vec{r}_1 - \vec{r}_2|) + \zeta(\vec{r}_1, \vec{r}_2) - \xi_{\text{is}}(r_1) \xi_{\text{is}}(r_2)] \frac{\phi(Z_1) \phi(Z_2)}{\phi_0^2 V^2}. \quad (17)$$

The last term is simply  $\langle \Delta_j \rangle^2$ . The terms in  $\xi_{\text{is}}$  in equation (16) integrate to zero. The  $\zeta$  term involves all three objects, so at the level of approximation in equation (2), we can assume  $\phi(Z_1) = \phi(Z_2) = \phi_0$ . However, the  $\xi_{ii}$  term represents correlated imaging galaxies that are uncorrelated with the spectroscopic object, so they may have  $Z$  far from 0 and  $\phi(Z) \neq \phi_0$ . Integrating over  $Z$  will yield the angular correlation of the imaging catalog. Indeed, for  $Z \neq 0$ , the angular separations implicit in our definition of the transverse coordinate  $\vec{R}$  correspond to different physical scales. Doing this correctly again yields the angular correlation function. We thus simplify equation (17) to

$$V_{\text{cl}}(\Delta_j) = V_{2\text{pt}} + V_{3\text{pt}} \quad (18) \\ V_{2\text{pt}} \equiv \left( \frac{\bar{n}}{\phi_0 V} \right)^2 \int d^2 R_1 \int d^2 R_2 G(R_1) G(R_2) w_{ii}(|\vec{R}_1 - \vec{R}_2|) \\ V_{3\text{pt}} \equiv \int \frac{d^3 r_1}{V} \int \frac{d^3 r_2}{V} G(R_1) G(R_2) [\zeta(\vec{r}_1, \vec{r}_2) - \xi_{\text{is}}(r_1) \xi_{\text{is}}(r_2)]$$

The first term can be done quickly by Fourier methods, as can the second if one adopts the hierarchical ansatz (Groth & Peebles 1977) to write  $\zeta(\vec{r}_1, \vec{r}_2) = Q\{\xi_{\text{is}}(r_1) \xi_{\text{is}}(r_2) + [\xi_{\text{is}}(r_1) + \xi_{\text{is}}(r_2)] \xi_{ii}(|\vec{r}_1 - \vec{r}_2|)\}$ . The two-dimensional Fourier transform of  $G(R)$  is

$$\int d^2 R e^{i\vec{k} \cdot \vec{R}} G(R) = 4 \int_0^\infty dr r \sin(kr) W(r) \quad (19)$$

Note that while  $V_{3\text{pt}}$  is a contribution to the variance about the mean  $\Delta_j$ , it is not necessarily noise! Much of it *is* the density around the particular object, which of course has scatter from the mean.

The shot noise or Poisson contribution to the variance is based on the expected counts, including clustering, which are  $\bar{n}[1 + w_{is}(R)]$ . The variance is

$$V_{\text{sn}}(\Delta_j) = \int d^2R \left[ \frac{G(R)}{\phi_0 V} \right]^2 \bar{n}[1 + w_{is}(R)]. \quad (20)$$

We refer to these two terms as the homogeneous and clustered shot noise, respectively.

For the Gaussian window (eq. [12]), the homogenous shot noise becomes  $2a^2\bar{n}/(V\phi_0)^2$ . Figure 1 shows the contribution per radial bin to the variance in the shot noise. Essentially all of the shot noise arises at  $R < 1.5a$ ; in other words, the fact that one is subtracting the background with a region at moderate radius rather than the entire sample adds little extra noise to the density estimator.

The four above contributions— $V_{2\text{pt}}$ ,  $V_{3\text{pt}}$ , homogeneous shot noise, and clustered shot noise—all have different scalings with the depth of the survey, the size of the window, and the clustering strength. For a typical survey thickness  $L \equiv \bar{n}/\phi_0$  and a typical window radius  $a$ , the contributions to the variance are roughly  $(L/20a)\Delta$ ,  $\Delta^2$ ,  $(L/125a)(1/a^3\phi_0)$ , and  $(\Delta/32)(1/a^3\phi_0)$ , respectively. The numerical coefficients are for illustration only<sup>1</sup>. The 2-point clustering term dominates on large scales; the clustered shot-noise on small scales.

The process of averaging many  $\Delta_j$  into a mean  $\Delta$  introduces additional error terms from the correlations of the spectroscopic galaxy positions, including contributions from the four-point correlation function. This is not surprising because  $\Delta$  is an integral of  $w_{is}(R)$ , whose covariance normally involves the four-point function. Neglecting these new terms in favor of the equations above corresponds to the assumption that the shot noise of the spectroscopic sample exceeds its clustering (see the expansions of) [Ber94, Ham97]. This is often a good assumption on small scales, particularly if one is considering only a small subset of the spectroscopic sample.

A crucial assumption of the analysis is that the objects that are uncorrelated but in close projection with the spectroscopic object are statistically identical to those in other parts of the sky. Magnification from weak lensing can violate this assumption in principle (see, e.g.,) [Val97]. Moreover, selection biases in cluster catalogs owing to superposition of unrelated structures (see, e.g.,) [Val01] are not reduced by this method.

## 2.6. From Kernels to Windows

If we are given  $G(R)$ , we can use the Abel integral to find the corresponding  $W(r)$ :

$$W(r) = \frac{1}{r} \int_0^r dR \frac{RG(R)}{\sqrt{r^2 - R^2}}. \quad (21)$$

If  $W(r)$  is to be bounded and  $V \neq 0$ ,  $G(R)$  must have an asymptotic form of  $-V/2\pi^2 R^3$ . In particular, this means that one cannot have  $G = 0$  for all large radii, thereby avoiding summations over large pair separations.

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<sup>1</sup>The numerical factors are computed for the Gaussian window with the assumptions of a power-law correlation function scaling as  $r^{-1.8}$  with uniform bias for all galaxies, an  $\alpha = -0.9$  Schechter luminosity function with imaging catalog luminosity cuts between  $0.4L^*$  and  $2.5L^*$ , the hierarchical form of  $\zeta$  with  $Q = 1.3$ , and a non-expanding cosmology (Euclidean metric, no K-corrections).

## 2.7. Other Windows

As one might expect, the spherical tophat is easily calculated but poorly behaved. Taking  $W(r) = 1$  for  $r < b$ , we find

$$G(R) = \begin{cases} 1 & R < b \\ \frac{2}{\pi} \left[ \sin^{-1} \frac{b}{R} - \frac{b}{\sqrt{R^2 - b^2}} \right] & R > b \end{cases} \quad (22)$$

This diverges as  $R \rightarrow b$  from above. The singularity is integrable in  $\Delta$  but divergent in the shot noise, making the tophat a poor choice.

Another simple choice that retains the advantage that  $W = 0$  for  $r > b$  is  $W(r) = 1 - r^2/b^2$  for  $r < b$ . We find

$$F(R) = \begin{cases} \frac{R^2}{2} - \frac{3R^4}{8b^2} & R < b \\ \frac{2}{\pi} \left[ \left( -\frac{b}{4} + \frac{3R^2}{8b} \right) \sqrt{R^2 - b^2} + \left( \frac{R^2}{2} - \frac{3R^4}{8b^2} \right) \sin^{-1} \frac{b}{R} \right] & R > b \end{cases} \quad (23)$$

and

$$G(R) = \begin{cases} 1 - \frac{3R^2}{2b^2} & R < b \\ \frac{2}{\pi} \left[ \frac{3\sqrt{R^2 - b^2}}{2b} + \left( 1 - \frac{3R^2}{2b^2} \right) \sin^{-1} \frac{b}{R} \right] & R > b \end{cases} \quad (24)$$

This is continuous at  $R = b$  but has somewhat worse shot noise properties than the Gaussian window.

## 3. Conclusions

I have presented a model-independent estimator for the spherically averaged overdensity of imaging catalog objects around spectroscopic objects. The method is simple to apply; one can view it as an alteration of standard background subtraction methods to yield spherical apertures rather than cylindrical ones. The method does not require an explicit inversion of the angular cross-correlation function to a spatial correlation function, although clearly if one finds oneself measuring the densities in multiple apertures of different sizes, one is effectively reverting back to an inversion method. The primary advantages of the new method compared to integrating the output of an inversion (i.e., computing eq. [3] and then eq. [4]) are that one does not need a smoothing prior and that the statistic can be applied to each spectroscopic galaxy independently (eq. [10]), leaving one free to sum the results *post facto* across as many spectroscopic subsamples as one desires. Like other angular methods, the new density estimator is unaffected by redshift distortions, which gives it an advantage on small scales over density estimation from spectroscopic catalogs.

Past work has assumed power-law correlation functions to recover spatial densities. This allows one to achieve higher signal-to-noise ratio and hence is a better choice for some applications. However, when probing the dependence of galaxy properties on small-scale environment, the model independence of the density estimator presented here is a valuable advantage. With today's large surveys (e.g., Y00,2dF), statistical precision is sometimes less precious than systematic control.

While it is clear that one can profitably consider the dependence of average density on the properties of the spectroscopic objects (Hogg et al. 2002), it is worth pointing out that one can also derive densities for subdivisions of the imaging catalog. For example, one can probe the color and/or luminosity distribution of objects within a spherical aperture of a particular set of spectroscopic objects (e.g., galaxies or clusters). A

speculative application would be to couple this approach to galaxy-galaxy weak lensing mass estimates. If one had a mass estimate for each imaging object, assuming the spectroscopic redshift, then one could find the masses of objects that are correlated with the spectroscopic tracer.

I thank David Hogg, Jon Loveday, Tim McKay, Ann Zabludoff, and Dennis Zaritsky for useful discussions. D.J.E. was supported by National Science Foundation (NSF) grant AST-0098577 and by a Alfred P. Sloan Research Fellowship.

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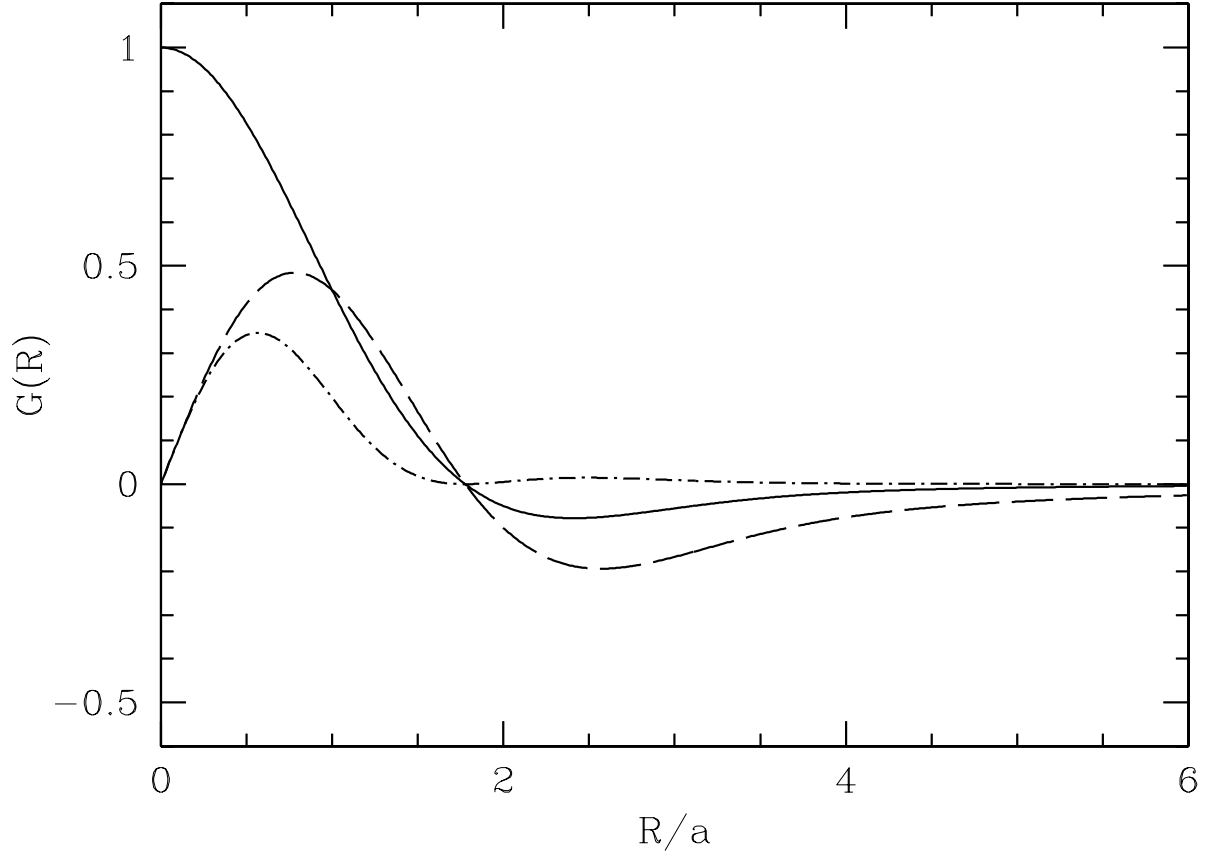


Fig. 1.— (*solid line*) The integration kernel  $G(R)$  for the Gaussian window. (*dashed line*)  $RG(R)$ , which is the weight per radial bin for a uniform background. (*dot-dashed line*)  $RG^2(R)$ , which is the shot noise variance per radial bin for a uniform distribution.